Solution of a Single Nonlinear (Implicit) Algebraic Equation with POLYMATH and MATLAB, Parametric Studies with MATLAB

A single nonlinear equation can be written in the form

$$f(x) = 0$$

where f is a function and x is the unknown. Additional explicit equations may also be included.

Typical examples belonging to this category include:

❖Solving various equations of state for molar volume and/or compressibility factor

❖Bubble point, dew point and isothermal flash calculations for ideal multi-component mixtures

❖Calculation of adiabatic flame temperature in combustion.

❖ Calculation of the flow rate in a pipeline.

Calculation of the Flow Rate in A Pipeline

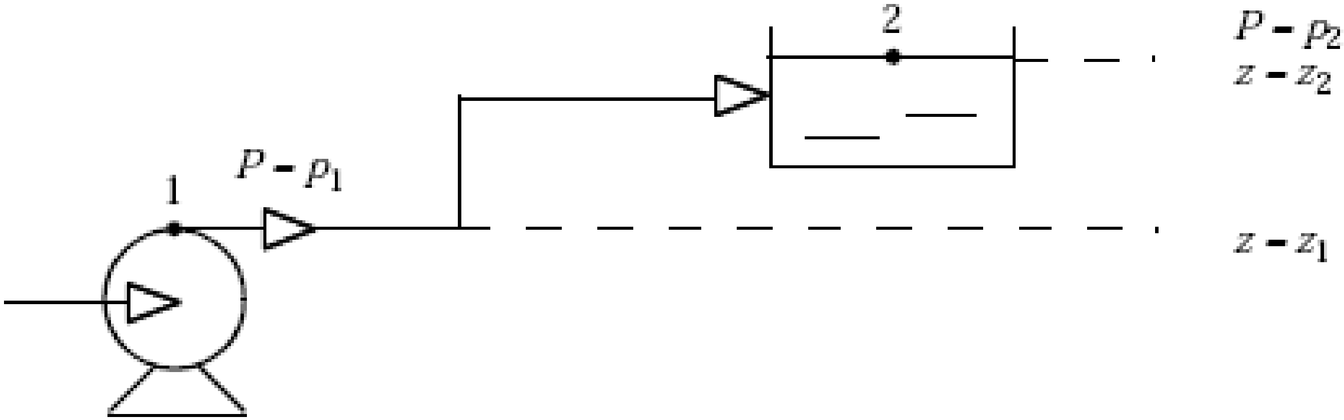


Figure 4–9 Pipeline at Steady State

Consider the pipeline which delivers water at constant temperature ***T* = 60 °F** from point 1 where the pressure is **p_1 = 150 psig** and the elevation is **z_1 = 0** ft to point 2 where the pressure is atmospheric and the elevation is **z_2 = 300 ft**.

Calculation of the Flow Rate in A Pipeline - Assignments

(a) Calculate, using POLYMATH, the flow rate q (in gal/min) for a pipeline with effective length of ***L* = 1000 ft** and made of nominal **8-inch diameter** schedule 40 commercial steel pipe. (Solution: *v* = 11.61 ft/s, *gpm* = 1811 gal/min)

(b) Use MATLAB to calculate the flow velocities in ft/s and flow rates in gal/min for pipelines at 60 °F with effective lengths of ***L* = 500, 1000, … 10000 ft** and made of nominal **4, 5, 6 and 8-inch** schedule 40 commercial steel pipe.

Present the results in tabular form and prepare plots of flow velocity *v* versus *D* and *L* and flow rate *q* versus *D* and *L*.

Calculation of the Flow Rate in A Pipeline – Model Equations

The general mechanical energy balance on an incompressible liquid applied to this case yields

$$-\frac{1}{2}v^2 + g\Delta z + \frac{g_c \Delta P}{\rho} + 2\frac{f_F L v^2}{D} = 0$$

where v is the flow velocity in ft/s, g is the acceleration of gravity given by g = 32.174 ft/s^2, $\Delta z = z2 - z1$ is the difference in elevation (ft), g_c is a conversion factor (in English units g_c = 32.174 ft·lbm/lbf·s^2), $\Delta P = P_2 - P_1$ is the difference in pressure lbm/ft^2), fF is the Fanning friction factor, L is the length of the pipe (ft) and $D$is the inside diameter of the pipe (ft).

$$f_F = \frac{16}{\text{Re}}$$

Laminar flow

Turbulent flow

$$f_F = 1/16\left\{\log\left[\frac{\varepsilon/D}{3.7} - \frac{5.02}{Re}\log\left(\frac{\varepsilon/D}{3.7} + \frac{14.5}{Re}\right)\right]\right\}^2$$

ε (pipe roughness) = 0.00015 ft for commercial steel pipes

Calculation of the Flow Rate in A Pipeline – Model Equations

The flow velocity in the pipeline can be converted to flow rate by multiplying it by the cross section are of the pipe, the density of water (7.481 gal/ft^3), and factor (60 s/min). Thus *q* has units of (gal/min). The inside diameters (*D*) of nominal 4, 5, 6 and 8-inch schedule 40 commercial steel pipes are 4.026 in, 5.047 in, 6.065 in and 7.981 in, respectively.

The density and viscosity of the water can be calculated from the equations:

$$\rho = 62.122 + 0.0122\,T - 1.54\times10^{-4}\,T^2 + 2.65\times10^{-7}\,T^3 - 2.24\times10^{-10}\,T^4$$

$$\ln\mu = -11.0318 + \frac{1057.51}{T + 214.624}$$

where T is in °F, ρ is in lb_m/ft^3, and μ is in $lb_m/ft\cdot s$.

Calculation of the Flow Rate in A Pipeline – POLYMATH Code

POLYMATH 6.10 Educational Release - [Nonlinear Equations Solver]

File Program Edit Format Problem Examples Window Help

safenewt Graph

Nonlinear Equations: 1 | Auxiliary Equations: 13 | Ready for solution

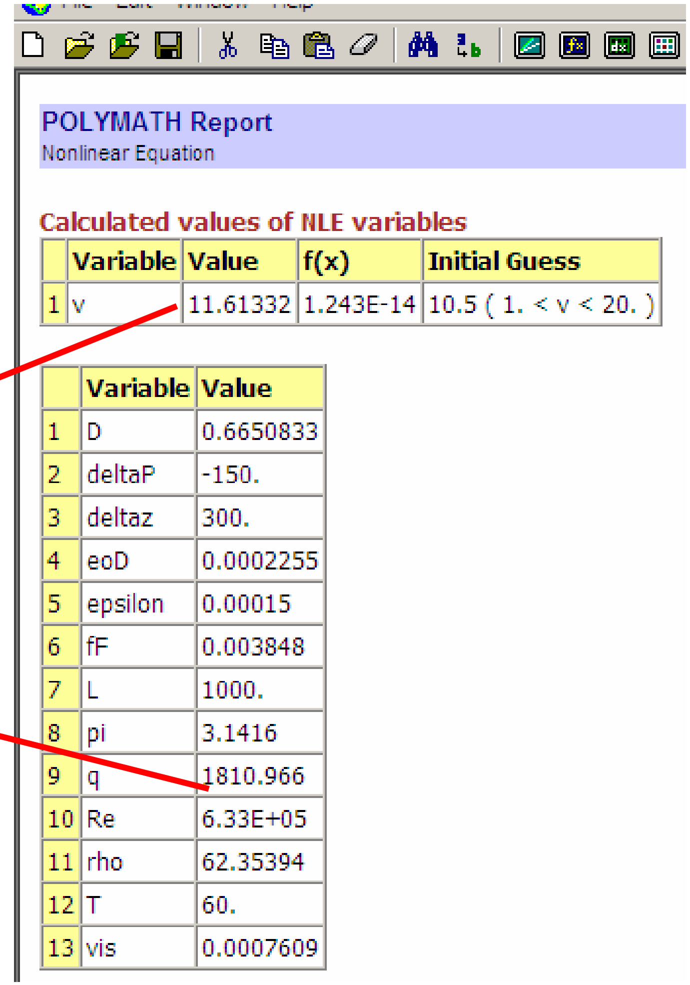
```
f(v) = v - sqrt((32.174 * deltaz + deltaP * 144 * 32.174 / rho) / (0.5 - 2 * fF * L / D)) # Flow velocity (ft/s)
fF = If (Re < 2100) Then (16 / Re) Else (1 / (16 * (log(eoD / 3.7 - 5.02 * log(eoD / 3.7 + 14.5 / Re) / Re)) ^ 2))
# Fanning friction factor (dimensionless)
eoD = epsilon / D # Pipe roughness to diameter ratio (dimensionless)
Re = D * v * rho / vis # Reynolds number (dimesionless)
deltaz = 300 # Elevation difference (ft)
deltaP = -150 # Pressure difference (psi)
T = 60 # Temperature (deg F)
L = 1000 # Effective length of pipe (ft)
D = 7.981 / 12 # Inside diameter of pipe (ft)
pi = 3.1416 # The constant pi
epsilon = 0.00015 # Surface rougness of the pipe (ft)
rho = 62.122 + T * (0.0122 + T * (-1.54e-4 + T * (2.65e-7 - T * 2.24e-10))) # Fluid density (lb/cu. ft.)
vis = exp(-11.0318 + 1057.51 / (T + 214.624)) # Fluid viscosity (lbm/ft-s)
q = v * pi * D ^ 2 / 4*7.481 * 60 # Flow rate (gal/min)
v(min) = 1
v(max) = 20
```

Note syntax of the implicit algebraic equation and the solution range provided

The POLYMATH code provides complete and clear documentation of the model

Calculation of the Flow Rate in A Pipeline – POLYMATH Solution

Calculate the flow rate q (in gal/min) for a pipeline with effective length of $L = 1000$ ft and made of nominal 8-inch diameter schedule 40 commercial steel pipe. (Solution: $v = 11.61$ ft/s, $gpm = 1811$ gal/min)



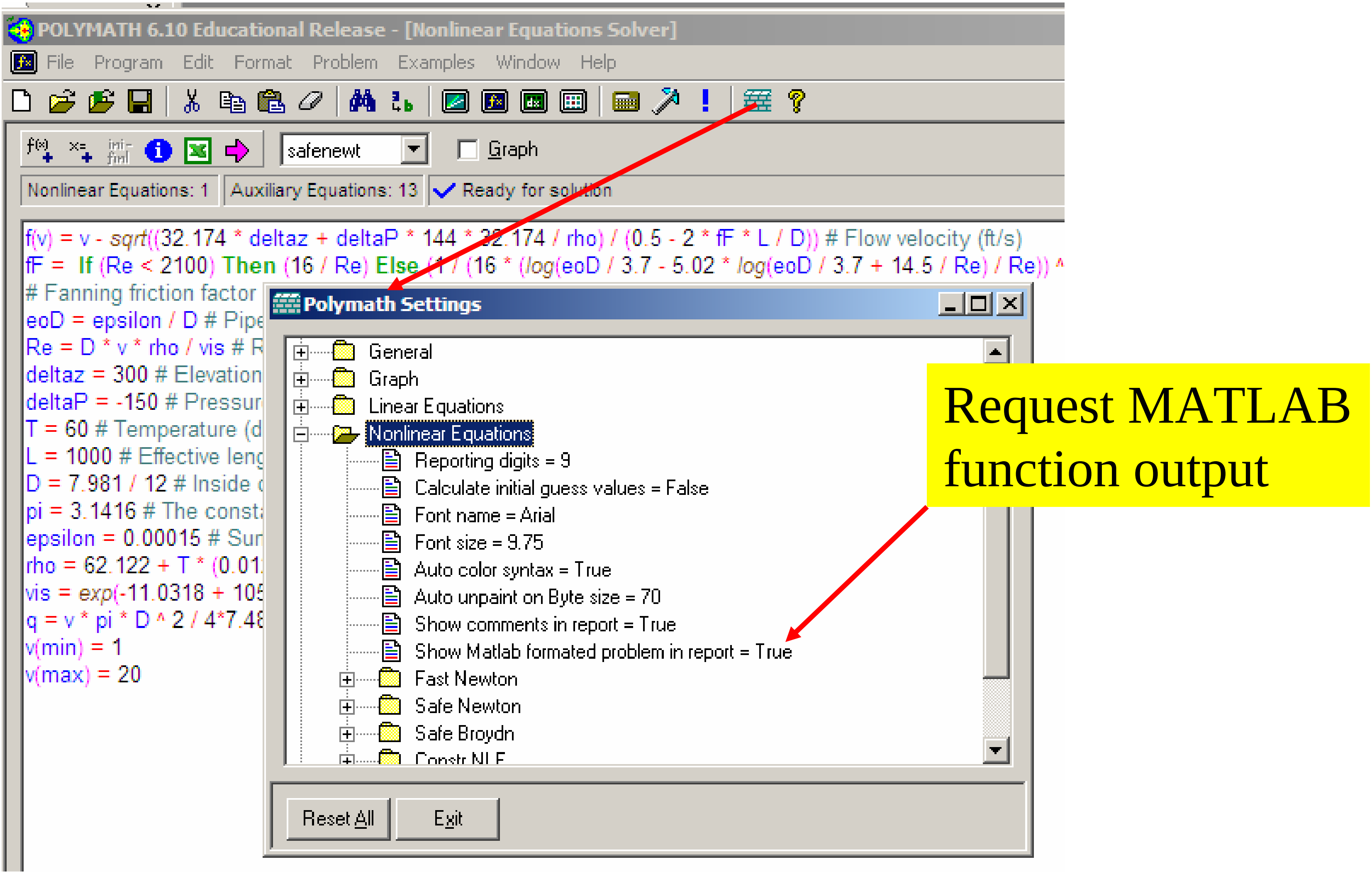
POLYMATH Report
Nonlinear Equation

Calculated values of NLE variables

	Variable	Value	f(x)	Initial Guess
1	v	11.61332	1.243E-14	10.5 (1. < v < 20.)

	Variable	Value
1	D	0.6650833
2	deltaP	-150.
3	deltaz	300.
4	eoD	0.0002255
5	epsilon	0.00015
6	fF	0.003848
7	L	1000.
8	pi	3.1416
9	q	1810.966
10	Re	6.33E+05
11	rho	62.35394
12	T	60.
13	vis	0.0007609

Calculation of the Flow Rate in A Pipeline– Converting POLYMATH Code to a MATLAB Function



Calculation of the Flow Rate in A Pipeline – A MATLAB Function generated by POLYMATH

Line	Command, %Comment
1	xguess = 10.5 ;
2	function fv = NLEfun(v);
3	T = 60; %Temperature (deg F)
4	epsilon = 0.00015; %Surface rougness of the pipe (ft)
5	rho = 62.122 + T * (0.0122 + T * (-1.54e-4 + T * (2.65e-7 - T * 2.24e-10)));%Fluid density (lb/cu. ft.)
6	deltaz = 300; %Elevation difference (ft)
7	deltaP = -150; %Pressure difference (psi)
8	vis = exp(-11.0318 + 1057.51 / (T + 214.624)); %Fluid viscosity (lbm/ft-s)
9	L = 1000; %Effective length of pipe (ft)
10	D = 7.981 / 12; %Inside diameter of pipe (ft)
11	pi = 3.1416; %The constant pi
12	eoD = epsilon / D; %Pipe roughness to diameter ratio (dimensionless)
13	Re = D * v * rho / vis; %Reynolds number (dimensionless)
14	if (Re < 2100) %Fanning friction factor (dimensionless)
15	fF = (16 / Re) ;
16	else
17	fF = (1 / (16 * (log10(eoD / 3.7 - 5.02 * log10(eoD / 3.7 + 14.5 / Re) / Re)) ^ 2));
18	end
19	q = v * pi * D ^ 2 / 4; %Flow rate (sq. ft./s)
20	gpm = q * 7.481 * 60; %Flow rate (gpm)
21	fv = v - sqrt((32.174 * deltaz + deltaP * 144 * 32.174 / rho) / (0.5 - 2 * fF * L / D));%Flow velocity (ft/s)

Does not belong to the function

Note that POLYMATH reordered the equations and changed the syntax

Template for Solving One Nonlinear Algebraic Equation*

Line	Command, %Comment
1	function % Insert here your file name after function (Use Alphanumberic name only)
2	clear, clc, format short g, format compact
3	xguess= % Replace this line with the xguess line(s) from Polymath report.
4	disp('Variable values at the initial estimate');
5	disp([' Unknown value ' num2str(xguess) ' Function Value ' num2str(NLEfun(xguess))]);
6	xsolv=fzero(@NLEfun,xguess);
7	disp(' Variable values at the solution');
8	disp([' Unknown value ' num2str(xsolv) ' Function Value ' num2str(NLEfun(xsolv))]);
9	%- -
10	% Replace this and the following line with the function copied from the Polymath report
11	% Do not include the xguess line(s)

The MATLAB library function *fzero* is used to solve the nonlinear equation

*Available in the HELP section of POLYMATH

Modifying the Template for Parametric Runs

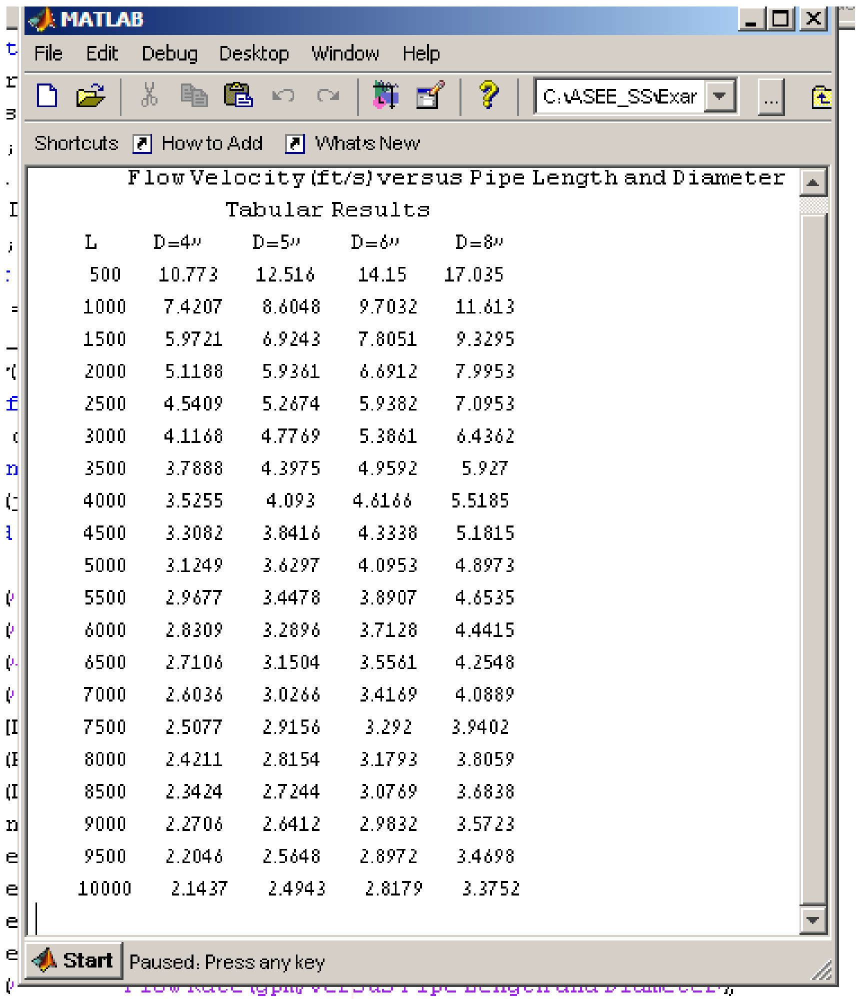
```
D_list=[4.026/12 5.047/12 6.065/12 7.981/12]; % Inside diameter of pipe (ft)
T = 60; %Temperature (deg. F)
for i = 1:4
    D = D_list(i);
    j=0;
    for L=500:500:10000
        j = j+1;
        L_list(j)=L; % Effective length of pipe (ft)
        [v(j,i),fval]=fzero(@NLEfun,[1 20],[],D,L,T);
        if abs(fval)>1e-10
            disp([' No Convergence for L = ' num2str(L) ' and D = ' num2str(D)]);
        end
        q(j,i) = v(j,i) * pi * D ^ 2 / 4* 7.481 * 60; %Flow rate (gpm)
    end
end
```

Pipe diameter loop

Pipe length loop

Input *D* and *L* as parameters into the function

Calculation of the Flow Rate in A Pipeline – Results of Parametric Runs



Flow Velocity (ft/s) versus Pipe Length and Diameter

Tabular Results

L	D=4"	D=5"	D=6"	D=8"
500	10.773	12.516	14.15	17.035
1000	7.4207	8.6048	9.7032	11.613
1500	5.9721	6.9243	7.8051	9.3295
2000	5.1188	5.9361	6.6912	7.9953
2500	4.5409	5.2674	5.9382	7.0953
3000	4.1168	4.7769	5.3861	6.4362
3500	3.7888	4.3975	4.9592	5.927
4000	3.5255	4.093	4.6166	5.5185
4500	3.3082	3.8416	4.3338	5.1815
5000	3.1249	3.6297	4.0953	4.8973
5500	2.9677	3.4478	3.8907	4.6535
6000	2.8309	3.2896	3.7128	4.4415
6500	2.7106	3.1504	3.5561	4.2548
7000	2.6036	3.0266	3.4169	4.0889
7500	2.5077	2.9156	3.292	3.9402
8000	2.4211	2.8154	3.1793	3.8059
8500	2.3424	2.7244	3.0769	3.6838
9000	2.2706	2.6412	2.9832	3.5723
9500	2.2046	2.5648	2.8972	3.4698
10000	2.1437	2.4943	2.8179	3.3752

Calculation of the Flow Rate in A Pipeline – Improving the Quality of Presentation by Exporting to Excel

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500	10.773	12.516	14.15	17.035
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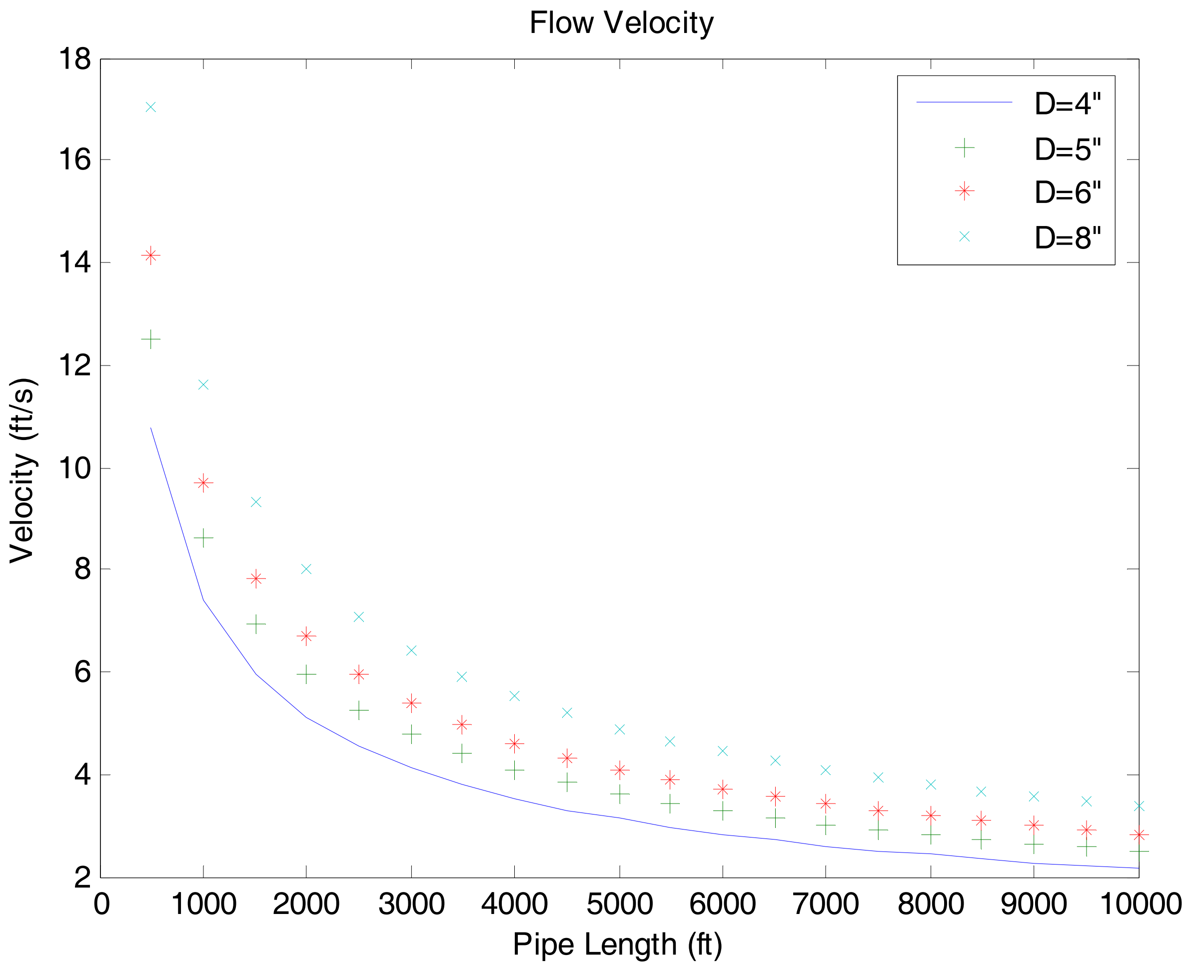
Calculation of the Flow Rate in A Pipeline – Results of Parametric Runs



Calculation of the Flow Rate in A Pipeline – Results of Parametric Runs